

PENETRATION OF A TARGET BY AN EXPLOSION  
IN THE IMPULSE-HYDRODYNAMIC MODE

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Recently, in the study of the effect of an explosion in solids the impulse mode for problems of hydrodynamics (see, for example, [1]) has been applied with the assumption that during the explosion action the medium behaves as an ideal incompressible fluid.

There exist several models for the effect of an explosion in which use is made of the impulse mode. One of these is a model for explosive cratering in soils [2]; in it the assumption is made that during the explosion all of the soil is in motion, while the crater boundary at the free surface is determined by equating its speed to a certain quantity  $v_0$ , referred to as the critical speed. Within the framework of this model, Vlasov considered a number of problems dealing with explosive cratering in soils and the penetration of a target. Lavrent'ev [3] made the assumption that the medium undergoing explosive loading is not in motion everywhere but only where the speed of the particles is larger than  $v_0$ . Where the particle speed is less than  $v_0$ , the medium is assumed to behave as a solid; the boundary separating these two regimes is the flow curve on which  $v = v_0$ . Such a model makes it possible to determine not only the width of the crater, but even its whole boundary. The quantity  $v_0$  characterizes the strength properties of the medium. The matter of how to determine  $v_0$  was considered in [4, 5]. Vlasov's model has come to be known as the fluid model (FM) and that of Lavrent'ev as the solid-fluid model (SFM).

In [6], within the SFM framework, a planar problem was considered and solved; this problem dealt with the penetration of a thin target by means of a surface line charge (LC) of constant thickness. Later on, other interesting problems relating to the penetration of a target (see, for example, [7-9]) were solved. However, up to this point, in connection with penetration problems, no comparisons have been made of the results obtained with the FM and SFM models with the experimental data. This is done in the present paper.

1. Description of the Experiments. As the explosive charge we used the detonating cord DC-A of radius  $2.5 \cdot 10^{-2}$  m, buried in the target to a depth of one diameter. The targets used were prepared from soil, from an alabaster-sand mixture, from foamy concrete, and from a plastic compound. The soil targets were made in the following way: two holes were dug (each in the form of a rectangular parallelepiped) close to one another, so that there remained connecting bridges between them varying in thickness from 0.05 m to 0.15 m; these latter served as the targets. Prior to the explosion the soil density was  $2.21 \cdot 10^3$  kg/m<sup>3</sup> and it had an 18.6% moisture content. The targets of sand and alabaster contained identical amounts of these components.

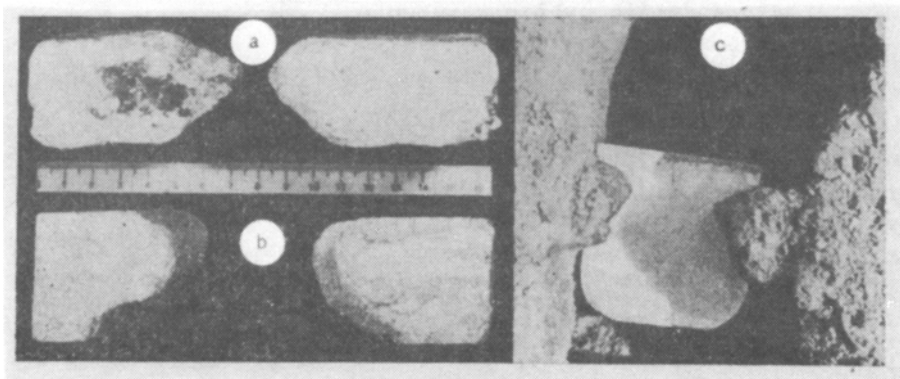


Fig. 1

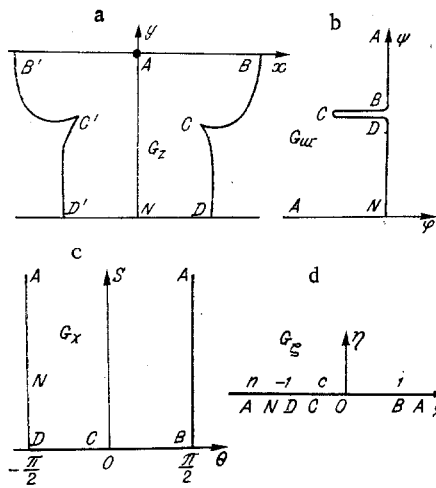


Fig. 2

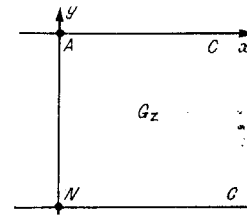


Fig. 3

Shown in Fig. 1a-c are the penetration profiles in the sand-alabaster, the foamy concrete, and the soil targets, respectively. In all cases the charge was placed under the upper free surface. Analyzing these profiles, we note that the penetration width varies substantially according to the height of the target: at first it narrows and then it widens; moreover, in the sand-alabaster and soil targets cusps are clearly discernible. In the course of our experiments it became clear that the upper part of the penetration arises from the ejection of material upwards, while the lower part arises from material being driven downward; moreover, no spall is observed in the case of targets of relatively small thickness (soil targets up to 0.12 m).

However, the penetration form described is evidently typical only for media not possessing sufficient elastic and plastic properties. Thus, for penetration in the plastic compound a large role is played by plastic deformations with the ejection of material being insignificant.

We now consider the penetration of a target by means of a single line charge within the framework of the hydrodynamic models.

2. Statement and Solution of a Problem Using the SFM Model. Let us assume that with the explosion of an infinitely long line charge, submerged to a depth of one diameter in a target of thickness  $H$ , the penetration cross section  $ABCDND'C'B'$  is formed (Fig. 2a, in which the plane  $z = x + iy$  is perpendicular to the charge axis, and  $BCD$  is a part of the flow curve, which starts out from the charge and divides at the point  $C$ ). It is well known [9] that a circular charge of radius  $r$  ( $r \ll H$ ), buried to a depth of one diameter, can be modeled as a dipole, whose moment  $M$  may be calculated from the formula  $M = 4\pi\Pi_0 r/\rho$ , where  $\Pi_0$  is the pressure impulse on the charge. Let the dipole be located at the point  $A$ .

We need to find the form and the dimensions of the penetration, i.e., the curve  $ABCDND'C'B'$ . In view of the symmetry with respect to the  $y$  axis, we need to consider only the right half of the physical domain, which we denote by  $G_z$ .

We introduce the dimensionless variables

$$z^* = z\sqrt{v_0/M}, w^* = w/\sqrt{Mv_0}, v^* = v/v_0. \quad (2.1)$$

Solution of the problem depends on the single parameter  $H^*$ . In what follows we shall omit the asterisk, indicating dimensionless variables. We solve the problem by the conformal mapping method employed in the theory of jets.

We introduce the hodograph function of the velocity  $\chi(z) = \theta + iS = i \ln(dw/dz)$ , where  $\theta$  is the angle between the velocity vector and the  $x$  axis, and  $S = \ln v$ .

For the functions  $w(z)$  and  $\chi(z)$  we have, on separate portions of the boundary of  $G_z$ , the boundary conditions

$$\begin{aligned} \varphi = 0, \theta = \pi/2 \text{ on } AB; \psi = \psi_1, S = 0 \text{ on } BCD; \\ \varphi = 0, \theta = -\pi/2 \text{ on } ND; \psi = 0, \theta = -\pi/2 \text{ on } NA \end{aligned}$$

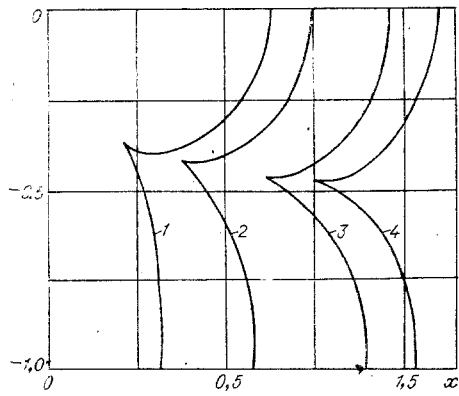


Fig. 4

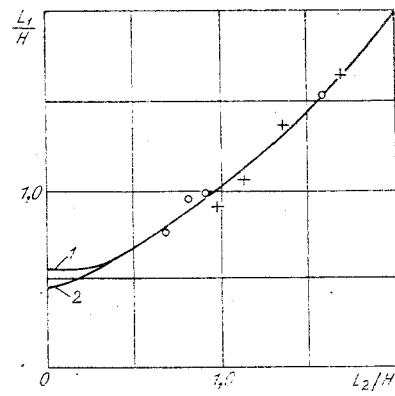


Fig. 5

( $\psi_1$  is a constant, so far not defined). By virtue of the latter conditions, the domain  $G_Z$  will correspond in the  $w$  and  $\chi$  planes to the domains  $G_w$  and  $G_\chi$  (Fig. 2b, c). We now map the half plane  $G_\zeta = \{\zeta: \text{Im} \zeta > 0\}$ ,  $\zeta = \xi + i\eta$  (Fig. 2d) conformally onto the domains  $G_w$  and  $G_\chi$  by means of the functions

$$w(\zeta) = \lambda \int_q^\zeta (\tau - d) / R(\tau, q) d\tau, \quad \chi(\zeta) = \arcsin \zeta,$$

where  $\lambda$  is a parameter, so far unknown;  $R(\tau, q) = \sqrt{(\tau^2 - 1)(\tau - q)}$ ; where for the root in question we take that branch which is positive for  $\tau > 1$ . Calculating  $w'(\zeta)$  and using the expression for  $\chi(z)$ , we obtain

$$z(\zeta) = \lambda \int_q^\zeta \frac{(\tau - d) \exp(i \arcsin \tau) d\tau}{R(\tau, q)}. \quad (2.2)$$

The expression for  $z(\zeta)$  depends on the three parameters ( $\lambda, d, q$ ). From the condition  $\text{Re} w(1) = 0$ , which can be written in the form

$$\int_{-1}^1 (\tau - d) / |R(\tau, q)| d\tau = 0,$$

we derive

$$d = d(q) = \frac{\int_{-1}^1 \frac{\tau d\tau}{|R(\tau, q)|}}{\int_{-1}^1 \frac{d\tau}{|R(\tau, q)|}}.$$

We calculate the parameter  $\lambda$  in the following way. It is known that if a dipole is present at the point  $z_0$ , then  $iM = 2\pi \lim_{z \rightarrow z_0} [(z - z_0) w'(z)]$ . Using l'Hospital's Rule and noting that the

point A of the domain  $G_Z$  corresponds to  $\xi = \infty$ , we have

$$1 = 2\pi i \lim_{\xi \rightarrow \infty} z' w^2 / w' = 2\pi \lambda^2 \lim_{\xi \rightarrow \infty} \left( \xi - \sqrt{\xi^2 - 1} \right) \left( \int_q^\xi \frac{(\tau - d) d\tau}{R(\tau, q)} \right)^2.$$

Evaluating the indeterminacy, we find  $4\pi\lambda^2 = 1$ , whence  $\lambda = \sqrt{1/4\pi}$ . With the aid of the equation  $\text{Im} z(1) = H$ , we determine  $q$  and, substituting its value into Eq. (2.2), we obtain, as  $\zeta \rightarrow \xi$ , the equation of the free boundary.

**3. Solution of the Problem Using the FM.** In this case, in accordance with the FM model, the domain  $G_Z$  has the form of an infinite half-strip (Fig. 3, considering only the right half of the physical plane). Since  $\varphi = 0$  on AC and NC,  $\psi = 0$  on AN, then, in the  $w$  plane, to the

domain  $G_z$  there corresponds the fourth quadrant, which is the domain  $G_w$ . The location of the point C in the  $w$  plane is determined in the process of obtaining the solution.

We introduce the dimensionless variables (2.1). Then, as was the case with the SFM, the solution of the problem depends on a single parameter, the thickness  $H$ . Mapping  $G_z$  onto  $G_w$ , and noting that at the point  $z = 0$  there is a dipole with unit moment, we have  $w(s) = i[\text{cth}(\pi z/2H)]/4H$ . Differentiating  $w(z)$  and equating the modulus of the resulting expression to one for  $y = 0$  and  $y = -H$ , we write formulas for determining the penetration halfwidth, from above  $L_1 = 2H \text{arsh}(\sqrt{\pi/8}/H)/\pi$  and from below  $L_2 = 2H \text{arch}(\sqrt{\pi/8}/H)/\pi$ .

4. Comparison of the Theoretical Results with Experimental Data. First of all, the form of the penetration obtained using the SFM agrees qualitatively in many ways with the form of the penetration obtained experimentally (Fig. 4, penetrations obtained with the SFM for  $H = 1, M = 1.75, 3, 7, 11$ ; results shown in curves 1-4, respectively). In addition, the mechanism for the formation of the penetration is the same: a portion of the penetration is formed at the expense of material ejected upward and a portion resulting from material driven downward.

We observe, first of all, the good qualitative agreement. Figure 5 represents the relationship  $L_1/H = f(L_2/H)$ . Curve 1 corresponds to the FM; curve 2 corresponds to the SFM; the experimental data +, O correspond to soil and plastic targets, respectively.

The relationship  $L_1/H = f(L_2/H)$ , shown graphically in Fig. 5, is identical for the FM and the SFM beginning with  $L_2/H \approx 0.4$ . What this says is that if one is required to determine only  $L_1$  and  $L_2$  and not the whole penetration boundary, it is then sufficient to use the simpler FM model.

We note, on the basis of these results, that the FM and the SFM agree with one another and with the experimental data sufficiently well, so that with suitable parametrization we have both qualitative and quantitative agreement.

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#### LITERATURE CITED

1. M. A. Lavrent'ev and B. V. Shabat, Problems of Hydrodynamics and Their Mathematical Modeling [in Russian], 2nd edn., Nauka, Moscow (1977).
2. O. E. Vlasov, Fundamentals of the Effects of an Explosion [in Russian], VIA, Moscow (1957).
3. M. A. Lavrent'ev, Variational Method in Boundary Value Problems for Equations of Elliptic Type [in Russian], Izd. Akad. Nauk SSSR, Moscow (1962).
4. V. M. Kuznetsov and É. B. Polyak, "Impulse-hydrodynamic schemes for cratering calculations with explosive line charges," Fiz.-Tekh. Probl. Razrab. Polezn. Iskop., No. 4 (1973).
5. V. A. Ivanov, "The effect of gravity on crater formation: thickness of ejecta and concentric basins," in: Proc. Lunar Sci. Conf., 7th, USA (1976).
6. V. M. Kuznetsov, "On an explosion on the surface of a plate," Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1962).
7. N. B. Il'inskii and R. B. Salimov, "On the solution of a boundary-value problem of explosion theory," Izv. Vyssh. Uchebn. Zaved., Mat., No. 6 (1975).
8. N. B. Il'inskii, A. V. Potashev, A. V. Rubinovskii, and P. A. Fishchenko, "Solution of some problems of explosion theory in the impulse-hydrodynamic mode," in: Proc. of Seminar on Boundary Value Problems [in Russian], Kazan Univ., No. 14 (1977).
9. A. V. Rubinovskii, "On the interaction of charges on the surface of a target in a jet hydrodynamic model," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 6 (1980).
10. N. B. Il'inskii, A. V. Potashev, A. V. Rubinovskii, and P. A. Fishchenko, "On two impulse-hydrodynamic models of explosive cratering," Dep. in VINITI, No. 2403-83, May 4 (1983).